Groundwater Remediation using Bayesian Information-Gap Decision Theory

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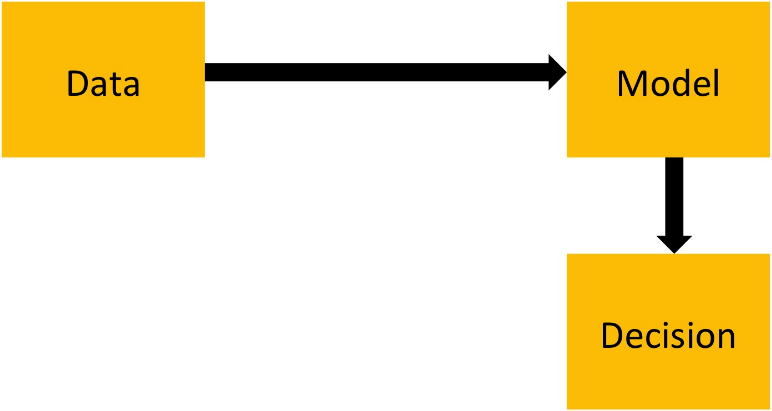
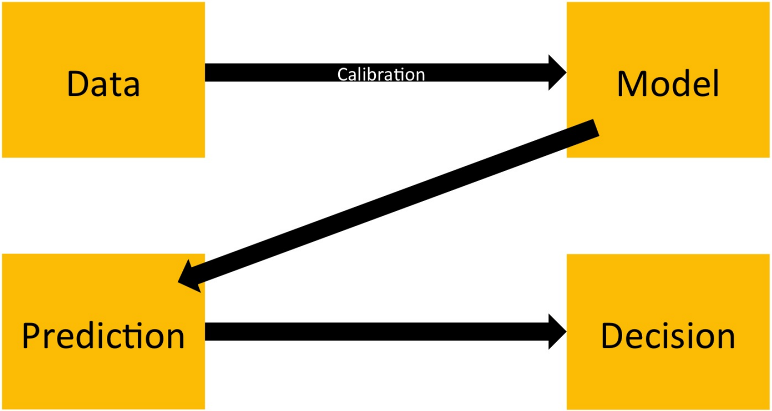
Data, Models & Decisions

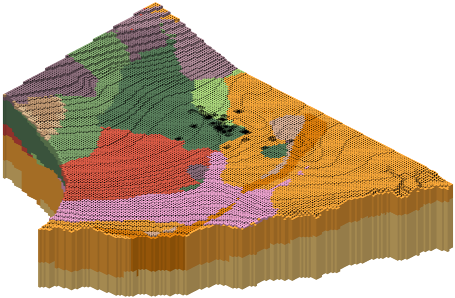


Data, Models & Decisions



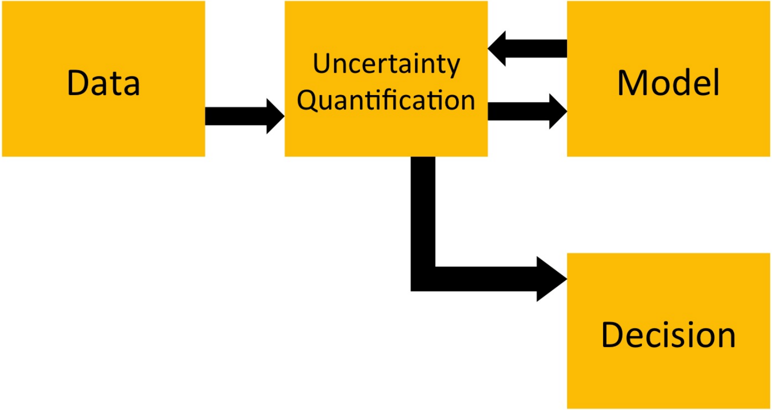
Models are often complex

- **Parameters**
  - heterogeneous, anisotropic permeability
  - heterogeneous specific storage
- **Model Physics**
  - saturated, confined aquifer
  - isothermal, incompressible
- **Discretization**
  - $\sim 10^6$ nodes
  - solved with obtuse codes



Shortcomings of this approach

- **Parameters**
  - small scale heterogeneity unrepresented
  - uncertainty not considered
- **Real Physics**
  - partially confined aquifer? vadose-zone effects?
  - boundary conditions? dual porosity? Noordbergum effect?
  - ...
  - even more complex once contaminant transport is added
- **Discretization**
  - relatively coarse discretization
  - numerical errors
- **Therefore**
  - Address the fact that we do not know the model parameters
  - Address the fact that we cannot represent the physics
  - Address the fact that we have uncertain observations


Data
Uncertainty Quantification
Model
Decision

The modeling solution

$$\mathbf{p} \rightarrow C_{\mathbf{p}}\left(\mathbf{x}, t\right)$$

Model parameters

Model

Model predictions

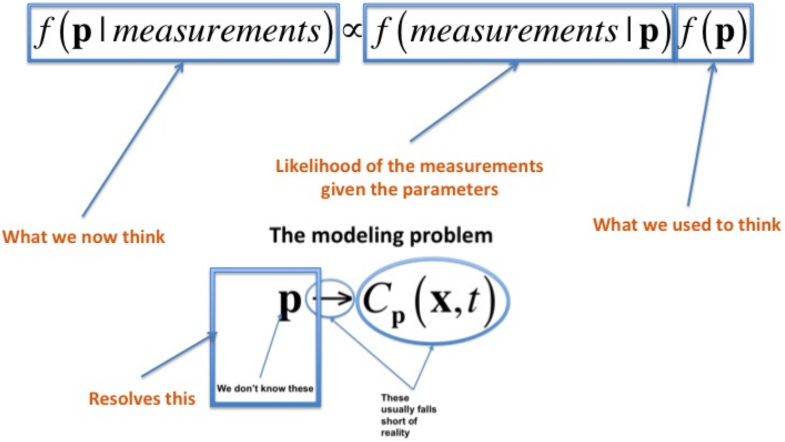
The modeling problem

$$\mathbf{p} \rightarrow C_{\mathbf{p}}(\mathbf{x}, t)$$

We don't know these

These usually fall short of reality

The Bayesian solution



The Bayesian problem

$$f\left(\mathbf{p} \mid measurements\right) \propto \boxed{f\left(measurements \mid \mathbf{p}\right)} f\left(\mathbf{p}\right)$$

We don't know this

The modeling problem

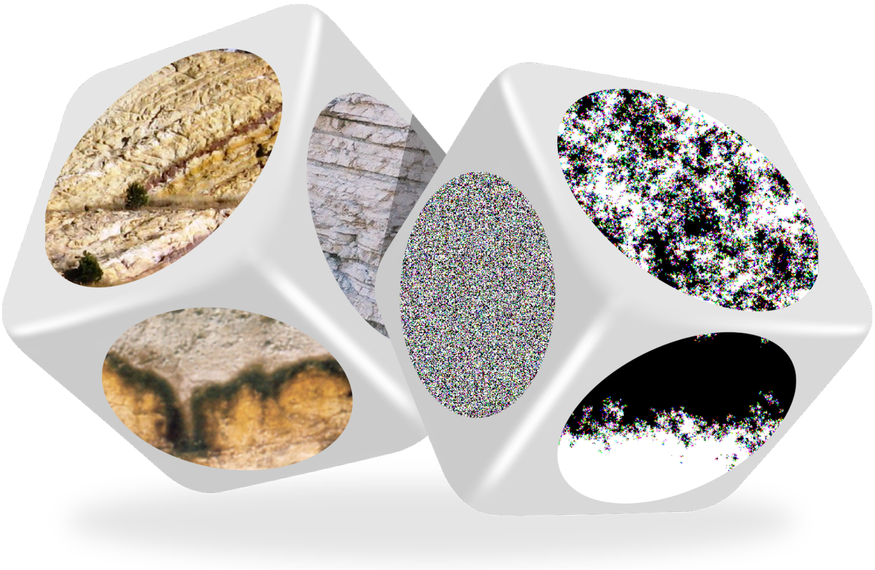
$$\boxed{\mathbf{p} \rightarrow C_{\mathbf{p}}\left(\mathbf{x}, t\right)}$$

Ignores this

We don't know the

These usually falls short of reality

Geologic dice



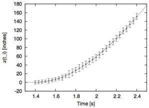
Bayesian problems

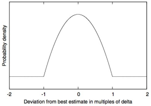
Bayes' theorem is mathematically rigorous, but its application in science and engineering is not always rigorous. There are two reasons for this:

- We can enumerate the possible outcomes of **dice-rolling**, but not all the possible outcomes for **real-world engineering problems**.
  - We cannot enumerate all possible permeability fields, and this just covers the first-order physics
  - NRC: 90% of court-mandated groundwater remediations fail, often due to unanticipated complexities
- We can precisely determine conditional probabilities for **coin-tossing**, but substantial uncertainty surrounds the conditional probabilities for **real-world engineering problems**.
  - It is observed that the water level in a well is $1750\ [m]$
  - Model A predicts a water level of $1749\ [m]$
  - Model B predicts a water level of $1748\ [m]$
  - What are the likelihoods of models A and B?
  - No one knows

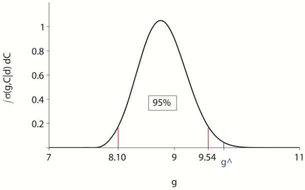
Bayesian problems: estimating gravitational acceleration

$$\frac{dv}{dt} = g - Cv^2, \ \frac{dz}{dt} = v, \ z(t_0) = v(t_0) = 0$$



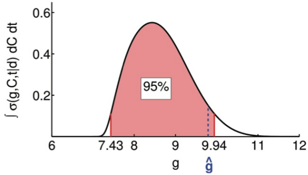


First try (g, C):



$$E(g|\mathbf{d}) = 8.82 \ [m/s^2]$$

Second try (g, C, t_0):

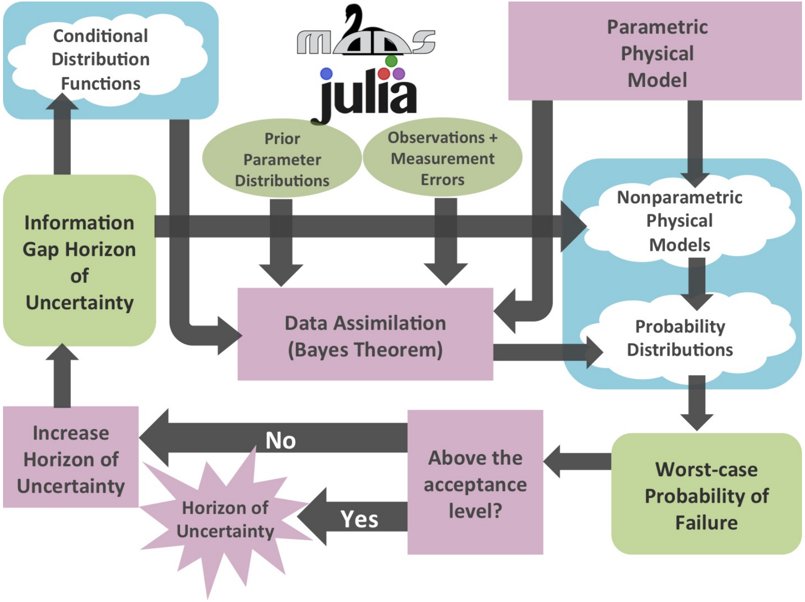


$$E(g|\mathbf{d}) = 8.64 \ [m/s^2]$$

Allmaras et al. *Estimating Parameters in Physical Models through Bayesian Inversion: A Complete Example*, SIAM Review (2013).

- **BIG** Decision Theory (DT)
- **B**ayesian-**I**nformation-**G**ap (**BIG**) Decision Theory (**DT**)
- Use Bayes theorem to
  - Assess parametric uncertainty
- Use information-gap decision theory to
  - Place the problem in a decision context
  - Consider the sides of the dice that the Bayesian analysis doesn't see

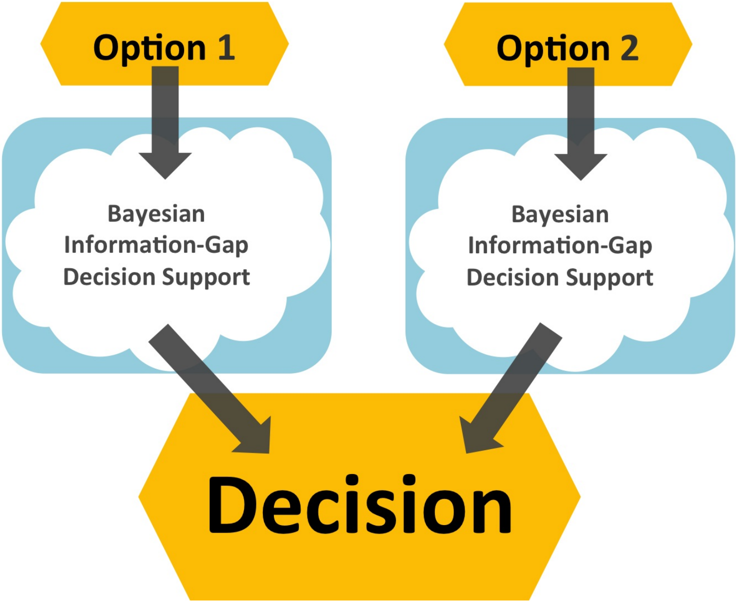
How to do BIG Decision Theory



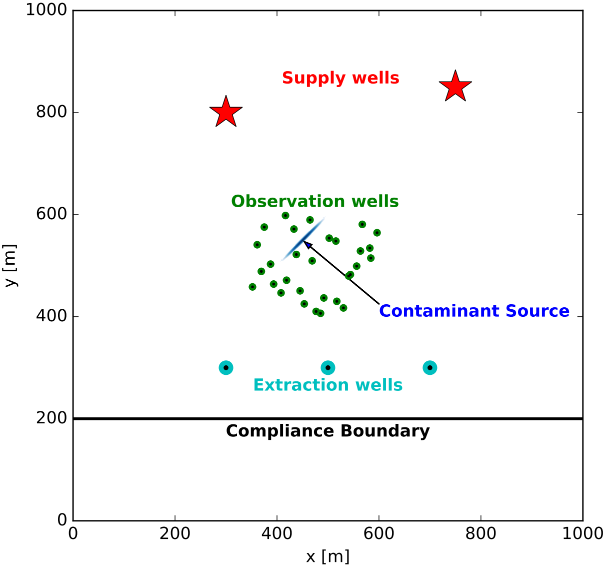
How to do BIG Decision Theory

Bayesian
Information-Gap
Decision Support

How to do BIG Decision Theory

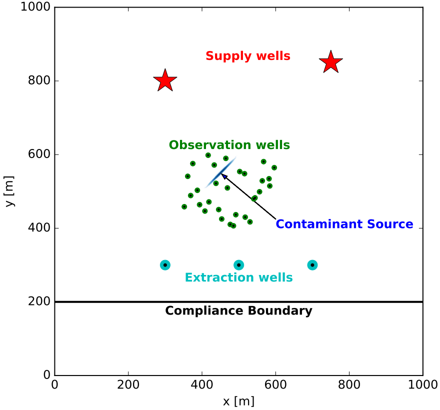


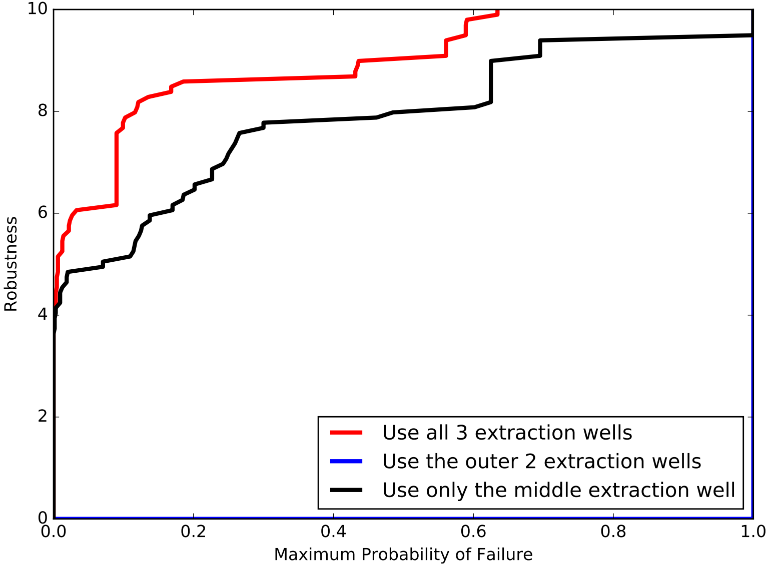
A representative scenario



Remedial options:

- Use the middle extraction well
- Use the outer extraction wells
- Use all extraction wells




Robustness
Maximum Probability of Failure
0.0
0.2
0.4
0.6
0.8
1.0
0
2
4
6
8
10
Use all 3 extraction wells
Use the outer 2 extraction wells
Use only the middle extraction well