RADIAL FLOW TO A PARTIALLY PENETRATING WELL WITH STORAGE IN AN ANISOTROPIC CONFINED AQUIFER

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ABSTRACT

Drawdowns generated by extracting water from a large diameter (e.g. water supply) well are affected by wellbore storage. We present an analytical solution in Laplace transformed space for drawdown in a uniform anisotropic aquifer caused by withdrawing water at a constant rate from a partially penetrating well with storage. The solution is back transformed into the time domain numerically. When the pumping well is fully penetrating our solution reduces to that of *Papadopulos et.al.* [1967]; when the pumping well is partially penetrating, has finite radius but lacks storage it coincides with a solution due to *Yang et.al.* [2006]. We use our solution to explore graphically the effects of partial penetration, wellbore storage and anisotropy on time evolutions of drawdown in the pumping well and in observation wells. Our note includes approximate criteria to help determine when partial penetration, wellbore storage and anisotropy have negligible effects on drawdown.

INTRODUCTION

When water is pumped from a large diameter (e.g. water supply) well drawdown in the surrounding aquifer is affected by temporal decline in wellbore storage. An analytical solution accounting for this effect under radial flow toward a fully penetrating well with storage was developed by *Papadopulos et.al.* [1967]. *Mathias et. al.* [2007] developed a corresponding solution in cartesian coordinates for a fully penetrating well in an vertically anisotropic aquifer. Their solution neglects effect of partial penetration and utilizes Mathieu functions in Laplace transformed space and numerical inversion of the result into the time domain. *Yang et.al.* [2006] extended the *Hantush* [1964] solution for finite diameter pumping well but neglected the storage capacity of the pumping well. In this note we develop solution for radial flow toward a partially penetrating well of finite diameter with storage. When the pumping well is fully penetrating our solution reduces to that of *Papadopulos et.al.* [1967]; when the pumping well is partially penetrating, has finite radius but lacks storage it coincides with a solution due to *Yang et.al.* [2006]. We use our solution to explore graphically the effects of partial penetration, wellbore storage and anisotropy on time evolutions of drawdown in the pumping well and in observation wells. Our note includes approximate criteria to help determine when partial penetration, wellbore storage and anisotropy have negligible effects on drawdown.

THEORY

Problem Definition

Consider a well of finite radius r_w that is in hydraulic contact with a surrounding confined aquifer at depths d through l below the top (Figure 1). The aquifer is horizontal and of infinite lateral extent with uniform thickness *b*, uniform hydraulic properties and anisotropy ratio

$K_D = K_z / K_r$ between vertical and horizontal hydraulic conductivities, K_z and K_r, respectively. Initially, drawdown $s(r,z,t)$ throughout the aquifer is uniformly zero where r is radial distance from the axis of the well, z is depth below the top of the aquifer and t is time. Starting at time $t = 0$ water is withdrawn from the pumping well at a constant volumetric rate Q. Consider the bottom of the well to be impermeable and ignore flow beneath it. Then drawdown distribution in space-time is controlled by

$$K_r\left(\frac{\partial^2 s}{\partial r^2}+\frac{1}{r}\frac{\partial s}{\partial r}\right)+K_z\frac{\partial^2 s}{\partial z^2}=S_s\frac{\partial s}{\partial t} \qquad 0 \le z < b \tag{1}$$

subject to

$$s(r,z,0)=s(\infty,z,t)=0 \qquad r \ge r_w \tag{2}$$

$$\frac{\partial s}{\partial z}=0 \quad \text{at } z=0 \ \& \ z=b \qquad r > r_w \tag{3}$$

$$2\pi(l-d)K_r r_w\left(\frac{\partial s}{\partial r}\right)_{r=r_w}-C_w\left(\frac{\partial s}{\partial t}\right)_{r=r_w}=-Q \qquad d < z < l \tag{4}$$

$$r_w\left(\frac{\partial s}{\partial r}\right)_{r=r_w}=0 \qquad 0 < z < d \quad \& \quad l < z < b \tag{5}$$

where C_w is wellbore storage coefficient (volume of water released from well storage per unit drawdown in it).

Solution in Laplace Space

We show in Appendix A that the Laplace transform of the solution, indicated by an overbar, is given by

$$\bar{s}\left(r_D, z_D, p_D\right)=\frac{Qt}{4\pi K_r b}\left\{\frac{2}{p_D}\frac{K_0\left(\phi_0\right)}{r_{wD}\phi_0 K_1\left(r_{wD}\phi_0\right)+\frac{C_{wD}}{2\left(l_D-d_D\right)}r_{wD}{}^2\left(\phi_0\right)^2 K_0\left(r_{wD}\phi_0\right)}\right. \tag{6}$$

$$\left.+\frac{4}{p_D\pi\left(l_D-d_d\right)}\sum_{n=1}^{\infty}\frac{K_0\left(\phi_n\right)\left[\sin\left(n\pi l_D\right)\text{-}\sin\left(n\pi d_D\right)\ \right]\cos\left(n\pi z_D\right)/n}{r_{wD}\phi_n K_1\left(r_{wD}\phi_n\right)+\frac{C_{wD}}{2\left(l_d-d_d\right)}r_{wD}{}^2\left(\phi_0\right)^2 K_0\left(r_{wD}\phi_n\right)}\right\}$$

where $r_D = r/b$, $z_D = z/b$, $p_D = pt$, $r_{wD} = r_w/r$, $l_D = l/b$, $d_D = d/b$, $C_{wD} = C_w/\left(\pi S r_w{}^2\right)$, $\phi_n = \sqrt{p_D/t_s + \beta^2 n^2 \pi^2}$, $t_s = \alpha_s t/r^2$, $\alpha_s = K_r/S_s$, $\beta = r_D K_D{}^{1/2}$, $K_D = K_z/K_r$ and K_0, K_1 are modified Bessel function of second kind and orders zero and one, respectively. A corresponding solution in the time domain is obtained through numerical inversion of the Laplace transform by means of an algorithm due to *Crump* [1976] as modified by *de Hoog et. al.* [1982]. Whereas standard inversion with respect to p is done over a time interval [0,t] , we do the inversion with respect to p_D over a unit dimensionless time (corresponding to $p_D{}^{-1}$) interval [0,1], regardless of what t_s is.

Vertically Averaged Drawdown to Observation Wells

Drawdown in a piezometer or observation well that penetrates the aquifer between dimensionless depths $z_{D1} = z_1/b$ and $z_{D2} = z_2/b$ at a dimensionless radial distance r_D from the pumping well (Figure 1) is obtained by averaging the point drawdown over this interval according to

$$s_{z_{D2}-z_{D1}}(r_D, t_s) = \frac{1}{z_{D2}-z_{D1}}\int_{z_{D1}}^{z_{D2}} s\left(r_D, z_D, t_s\right) dz_D . \tag{7}$$

Substituting (6) into (7) and evaluating the integral gives

$$\bar{s}=\frac{Qt}{4\pi K_r b}\left\{\frac{2}{p_D}\frac{K_0\left(\phi_0\right)}{r_{wD}\phi_0 K_1\left(r_{wD}\phi_0\right)+\frac{C_{wD}}{2\left(l_D-d_D\right)}r_{wD}{}^2\phi_0{}^2K_0\left(r_{wD}\phi_0\right)}\right.$$

$$\left.+\frac{4}{p_D\pi^2\left(l_D-d_d\right)(z_{D2}-z_{D1})}\sum_{n=1}^{\infty}\frac{K_0\left(\phi_n\right)\left[\sin\left(n\pi l_D\right)\text{-}\sin\left(n\pi d_D\right)\right]\left[\sin\left(n\pi z_{D2}\right)-\sin\left(n\pi z_{D1}\right)\right]/n^2}{r_{wD}\phi_n K_1\left(r_{wD}\phi_n\right)+\frac{C_{wD}}{2\left(l_d-d_d\right)}r_{wD}{}^2\phi_0{}^2K_0\left(r_{wD}\phi_n\right)}\right\}$$

(8)

Reduction to Solution of *Papadopoulos et.al.* [1967]

When the pumping well is fully penetrating $l_d=1$, $d_d=0$ and (6) reduces to the corresponding Laplace domain solution of *Papadopoulos et.al.* [1967],

$$\bar{s}\left(r_D,z_D,p_D\right)=\frac{Qt}{4\pi K_r b}\left\{\frac{2}{p_D}\frac{K_0\left(\sqrt{\frac{p_D}{t_s}}\right)}{r_{wD}\sqrt{\frac{p_D}{t_s}}K_1\left(r_{wD}\sqrt{\frac{p_D}{t_s}}\right)+\frac{C_{wD}}{2}r_{wD}{}^2\frac{p_D}{t_s}K_0\left(r_{wD}\sqrt{\frac{p_D}{t_s}}\right)}\right\}$$

(9)

Reduction to Solution's of *Yang et.al.* [2006], *Hantush* [1964] and *Theis* [1935]

When the pumping well has finite diameter $\left(r_w\neq 0\right)$ but has negligible or no wellbore storage $\left(C_{wD}\to 0\right)$, (6) reduces to Laplace domain solution of *Yang et. al*. [2006]

$$\bar{s}\left(r_D,z_D,p_D\right)=\frac{Qt}{4\pi K_r b}\left\{\frac{2}{p_D}\frac{K_0\left(\phi_0\right)}{r_{wD}\phi_0 K_1\left(r_{wD}\phi_0\right)}\right.$$

(10)

$$\left.+\frac{4}{p_D\pi\left(l_D-d_d\right)}\sum_{n=1}^{\infty}\frac{K_0\left(\phi_n\right)\left[\sin\left(n\pi l_D\right)\text{-}\sin\left(n\pi d_D\right)\ \right]\cos\left(n\pi z_D\right)/n}{r_{wD}\phi_n K_1\left(r_{wD}\phi_n\right)}\right\}$$

When the pumping well has small diameter $\left(r_w \to 0\right)$, considering that $xK_1(x) \to 1$ and $x^2 K_0(x) \to 0$ as $x \to 0$, (6) reduces to *Hantush*'s [1964] solution in Laplace space.

$$\overline{s}\left(r_D, z_D, p_D\right) = \frac{Qt}{4\pi K_r b}\left\{\frac{2}{p_D} K_0\left(\sqrt{\frac{p_D}{t_s}}\right) + \frac{4}{p_D \pi}\sum_{n=1}^{\infty}\frac{K_0\left(\phi\right)\left[\sin\left(n\pi l_D\right) - \sin\left(n\pi d_D\right)\ \right]\cos\left(n\pi z_D\right)}{n\left(l_D - d_D\right)}\right\} \ . \tag{11}$$

It is well established and easily verified here that the latter in turn reduces to the *Theis* [1935] solution in Laplace space when the pumping well becomes fully penetrating $\left(l_d = 1, d_d = 0\right)$,

$$\overline{s} = \frac{Qt}{4\pi K_r b}\left\{\frac{2}{p_D} K_0\left(\sqrt{\frac{p_D}{t_s}}\right)\right\} \tag{12}$$

3. RESULTS AND DISCUSSION

To investigate the effect of partial penetration, wellbore storage and anisotropy on drawdown we consider a pumping well of dimensionless radius $r_w / b = 0.02$. Unless we state otherwise, drawdown other than in the pumping well is evaluated at a point located midway between the top and bottom boundaries at $z / b = 0.5$ and at a dimensionless radial distance $r / b = 0.2$ from the axis of the pumping well.

Drawdown in pumping well

We start by considering pumping well penetrating the upper half ($d_D = 0.0, l_D = 0.5$) of an isotropic aquifer with $K_D = 1.0$. Figure 2 compares the variation of dimensionless drawdown $s_D\left(r_D, z_D, t_s\right) = \left(4\pi K_r b / Q\right) s\left(r, z, t\right)$ in the pumping well with dimensionless time t_s using different analytical solutions when $C_{wD} = 1.0\times10^2$. It is clear that disregarding partial penetration (a case represented in Figure 2 by the *Papadopulos et.al* [1967] and *Theis* [1935] solutions) underestimates drawdown in the pumping well. C_{wD}. At early time water is derived entirely from wellbore storage, rendering dimensionless drawdown linearly proportional to dimensionless time (forming a line of unit slope on log log scale) and our solution approaches that of *Papadopulos et.al* [1967]. At late dimensionless time effects of wellbore storage and finite radius of pumping well diminishes and our solution approaches that of *Hantush* [1964].

Figure 3 shows how dimensionless drawdown $s_D\left(r_D, z_D, t_s\right) = \left(4\pi K_r b / Q\right) s\left(r, z, t\right)$ in the pumping well varies with dimensionless time t_s for different values of dimensionless wellbore storage coefficient The larger is C_{wD}, the longer does wellbore storage impact drawdown in the pumping well. At smaller C_{wD} effects of wellbore storage is negligible and our solution reduces to those given by *Yang et.al* [2006].

Drawdown in an observation piezometer

First, we investigate the impact of dimensionless wellbore storage C_{wD} of the pumping well on drawdown observed at observation piezometer by setting $d_D = 0.0$, $l_D = 0.5$ and $K_D = 1.0$. Figure 4 presents the variation of dimensionless drawdown with dimensionless time at dimensionless vertical distance $z/b = 0.5$ below the top impermeable boundary and at dimensionless radial distance $r/b = 0.2$ from the pumping well. Effects of wellbore storage on drawdown decreases as the dimensionless wellbore storage decreases and for small values the dimensionless wellbore storage the time-drawdown curve reduces to those given by *Yang et al*. [2006] which in turn approaches to *Hantush* [1964]. In contrast with Figure 3, the straight line behavior in time drawdown curve diminishes with decrease in dimensionless wellbore storage C_{wD} of pumping well. This is because at the early time the drawdown at observation well is lumped response of water drawn from wellbore storage and those from radial flow component. The radial flow along the well has diluting effect on wellbore storage phenomenon.

To investigate the effect of partial penetration of the pumping well on drawdown at the observation well, C_{wD} is held constant ($C_{wD} = 1.0 \times 10^2$) and screen length $\left(l_d - d_d\right)$ is varied. Figure 5a depicts the dimensionless time-drawdown behavior with various degree of penetration of penetration. Figure 5a demonstrates that the effect of partial penetration is important especially when the bottom of pumping well screen is located vertically above the observation point. After the screen length is increased from top 50% to 100 % of thickness of confined aquifer, the effect of partial penetration on drawdown at observation diminishes.

Based on equation (6), the anisotropy is important only when the pumping well is partially penetrating. To investigate the effects of directional hydraulic conductivity (anisotropy) on time-drawdown behavior, we considered pumping well penetrating top 25 % of confined aquifer. Figure 5b depicts the time-drawdown for different anisotropy ratio assuming fixed wellbore storage $C_{wD} = 1.0 \times 10^2$. Figure 5b demonstrates that for the case where pumping well is partially penetrating the anisotropy will have impact on the all segments of the time drawdown curve. As the anisotropic ratio decrease meaning the hydraulic conductivity in vertical direction decreases, the less and less drawdown is observed at the observation piezometer.

Figure 6 depicts time drawdown behavior for different dimensionless radial distance between the pumping well and the observation point. The observation point is located along the center line of the confined aquifer ($z/b = 0.5$) and the wellbore storage is fixed ($C_{wD} = 1.0 \times 10^3$). As the radial distance from the pumping well increase the effect of wellbore storage and partial penetration on drawdown observed at observation piezometer diminishes and time-drawdown behavior approaches as given by *Theis* [1935] solution. Figure 6 shows that the critical distance after which the effects of wellbore storage and partial penetration are negligible on drawdown observed at observation piezometer is in this case $r/b > 6.3$.

Critical time for wellbore storage effects

It is intuitive that time during which the wellbore storage effects will be predominant in time-drawdown curve will be small for aquifers which have large transmissivity and/or small wellbore storage coefficients. By comparing type curves obtained from proposed analytical solution and corresponding *Yang et.al.* [2006] solution we found that for dimensionless time

$$t_s \geq 50 \frac{C_{wD} r_{wD}^{\ 2}}{\left(l_d - d_d\right)} \tag{13}$$

the difference in dimensionless drawdown is within 1 - 5 %.

Equation (13) will give corresponding limit on time as

$$t \geq \frac{50}{\pi} \frac{C_w}{\left(l_d - d_d\right) T} \tag{14}$$

It is to be noted that after comparing type curves of their solutions with those of *Theis* [1935] solution, *Papadopulos et.al.* [1967] suggested critical time as $t \geq \frac{25}{\pi} \frac{C_w}{T}$.

Critical distance for partial penetration and wellbore storage effects

The distance after which the partial penetration and wellbore storage effects will be negligible can serve as an important guideline for designing the locations of observation wells collecting pumping test data. To avoid the effect of wellbore storage on time drawdown, equation (14) needs to be satisfied which gives

$$r_D \geq \sqrt{50 \frac{C_{wD} \left(r_w / b\right)^2}{\left(l_d - d_d\right) t_s}} \tag{15}$$

Where t_s corresponds to the dimensionless time after which the wellbore storage effects are not desired in time drawdown curve. Smaller the t_s chosen farther the location of observation well would be needed. We suggest choosing $t_s = 1$ for practical purpose.

$$r \geq b\sqrt{50\frac{C_w}{\pi S\left(l_d - d_d\right)}} \tag{16}$$

After comparing type curves of their solution with corresponding *Theis* [1935] solution *Hantush* [1964] suggested following expression for the critical radial distance after which partial penetration effects are negligible

$$r \geq 1.5b\sqrt{K_z / K_r} \tag{17}$$

Critical radius will be determined when equation (16) and equation (17) are simultaneously satisfied. For wells with small wellbore storage and/or large transmissivity equation (17) will serve as more restrictive criterion where as for wells with large wellbore storage effects and/or small transmissivity the equation (16) will be more restrictive. Combining (16) and (17) together the criterion for critical radius becomes

$$r / b \geq \max\left\{1.5\sqrt{K_z / K_r}\ ,\ \sqrt{50\frac{C_w}{\pi S\left(l_d - d_d\right)}}\right\} \tag{18}$$

It can also be verified from figure 6 that critical radius for corresponding set parameters is $r / b = 6.3$.

4. SUMMARY AND CONCLUSION

A new analytical solution is developed for partially penetrating pumping well with wellbore storage in an infinite extent spatially uniform anisotropic aquifer. Our new solution generalizes those given by *Yang et.al*. [2006] and *Papadopulos et.al.* [1967] and reduces to that given by *Yang et.al*. [2006] if wellbore storage is negligible and to those by *Papadopulos et.al.* [1967], for fully penetrating wells with wellbore storage in an isotropic aquifer. The effect of partial penetration decreases with increasing distance from the pumping well whereas effect of wellbore storage diminishes with distance and time. After comparing type curves of our solution with corresponding *Yang et.al*. [2006] and *Theis* [1935] solution we propose approximate expressions for critical time and critical distance when the effects of partial penetration and wellbore storage are negligible.

Appendix A: Laplace transformed drawdown

Introducing a new variable $r' = r\left(K_z / K_r\right)^{1/2} = rK_D{}^{1/2}$ and taking Laplace transform of (1) – (5) gives

$$\frac{\partial^2 \overline{s}}{\partial r'^2} + \frac{1}{r'}\frac{\partial \overline{s}}{\partial r'} + \frac{\partial^2 \overline{s}}{\partial z^2} = \frac{S_s}{K_z} p\overline{s} \qquad 0 \le z < b \tag{A1}$$

subject to

$$\overline{s}\left(\infty, z, p\right) = 0 \tag{A2}$$

$$\frac{\partial \overline{s}}{\partial z} = 0 \quad \text{at } z = 0 \ \& \ z = b \qquad r > r_w \tag{A3}$$

$$2\pi\left(l-d\right)K_r r_w' \left(\frac{\partial \overline{s}}{\partial r'}\right)_{r'=r_w'} - C_w p\overline{s}_{r'=r_w'} = -\frac{Q}{p} \qquad d < z < l \tag{A4}$$

$$r_w' \left(\frac{\partial \overline{s}}{\partial r'}\right)_{r'=r_w'} = 0 \qquad 0 < z < d \quad \& \quad l < z < b \tag{A5}$$

Defining the Fourier cosine transform of $\bar{s}\left(r', z, p\right)$ as (*Churchill*, 1958, p.354-355)

$$f_c\left\{\bar{s}\left(r', z, p\right)\right\} = \bar{s}_c\left(r', n, p\right) = \int_0^b \bar{s}\left(r', z, p\right)\cos\left(n\pi z / b\right)\,\mathrm{d}z \qquad\qquad n = 0, 1, 2, ... \tag{A6}$$

with inverse

$$\bar{s}\left(r', z, p\right) = \frac{1}{b}\bar{s}_c\left(r', 0, p\right) + \frac{2}{b}\sum_{n=1}^{\infty}\bar{s}_c\left(r', n, p\right)\cos\left(n\pi z / b\right) \tag{A7}$$

implies that, by virtue of (A3),

$$f_c\left\{\frac{\partial^2 \bar{s}}{\partial z^2}\right\} = -\left(\frac{n\pi}{b}\right)^2 \bar{s}_c\left(r', n, p\right) + \left(-1\right)^n \frac{\partial \bar{s}\left(r', z, p\right)}{\partial z}\bigg|_{z=b} - \frac{\partial \bar{s}\left(r', z, p\right)}{\partial z}\bigg|_{z=0} = -\left(\frac{n\pi}{b}\right)^2 \bar{s}_c\left(r', n, p\right) \tag{A7}$$

Hence Fourier cosine transformation of (A1) – (A5) leads to

$$\frac{\partial^2 \bar{s}_c}{\partial r'^2} + \frac{1}{r'}\frac{\partial \bar{s}_c}{\partial r'} - \left[\frac{p}{K_z / S_s} + \left(\frac{n\pi}{b}\right)^2\right]\bar{s}_c = 0 \tag{A8}$$

$$\bar{s}_c\left(\infty, n, p\right) = 0 \tag{A9}$$

$$\begin{aligned} 2\pi\left(l-d\right)K_r r_w'\left(\frac{\partial \bar{s}_c}{\partial r}\right)_{r'=r_w'} - pC_w\left(\bar{s}_c\right)_{r'=r_w'} &= -\frac{Q}{p}\int_d^l \cos\left(n\pi z / b\right)dz \\ &= -\frac{Q}{p}\left(b / n\pi\right)\left[\sin\left(n\pi l / b\right) - \sin\left(n\pi d / b\right)\right] \end{aligned} \tag{A10}$$

The general solution of (A8) is

$$\bar{s}_c = AK_0\left(Nr'\right) + BI_0\left(Nr'\right) \tag{A11}$$

where $N^2 = \dfrac{p}{K_z / S_s} + \left(n\pi / b\right)^2$, I_0 and K_0 being modified Bessel functions of first and second kind, respectively, and of zero order. By virtue of (A9) $B = 0$. Substituting this and (A11) into (A10), noting that $\partial K_0\left(Nr'\right) / \partial r' = -NK_1\left(Nr'\right)$, solving for *A* and substituting back into (A11) yields

$$\bar{s}_c = \frac{Q}{p}\frac{(b/n\pi)\left[\sin(n\pi l/b)\text{-}\sin(n\pi d/b)\ \right]}{2\pi(l-d)K_r N r'_w K_1(N r'_w) + pC_w K_0(N r'_w)} K_0(N r'). \quad \text{(A12)}$$

Noting that $\lim_{n\to 0}\left[l\,\frac{\sin(n\pi l/b)}{n\pi l/b} - d\,\frac{\sin(n\pi d/b)}{n\pi d/b}\ \right] = l-d$ one gets

$$\bar{s}_c(0) = \frac{Q}{p}\frac{K_0\left(r'\sqrt{\frac{p}{K_z/S_s}}\right)}{2\pi K_r r'_w\sqrt{\frac{p}{K_z/S_s}}K_1\left(r'_w\sqrt{\frac{p}{K_z/S_s}}\right) + \frac{pC_w}{(l-d)}K_0\left(r'_w\sqrt{\frac{p}{K_z/S_s}}\right)}. \quad \text{(A13)}$$

This allows obtaining the inverse Fourier cosine transform of (A12),

$$\begin{aligned}\bar{s} &= \frac{1}{b}\frac{Q}{K_r p}\frac{K_0\left(r'\sqrt{\frac{p}{K_z/S_s}}\right)}{2\pi r'_w\sqrt{\frac{p}{K_z/S_s}}K_1\left(r'_w\sqrt{\frac{p}{K_z/S_s}}\right) + \frac{pC_w}{K_r(l-d)}K_0\left(r'_w\sqrt{\frac{p}{K_z/S_s}}\right)} \\ &\quad + \frac{2}{b}\frac{Q}{K_r p}\sum_{n=1}^{\infty}\frac{(b/n\pi)\left[\sin(n\pi l/b)\text{-}\sin(n\pi d/b)\ \right]}{2\pi(l-d)N r'_w K_1(N r'_w) + \frac{pC_w}{K_r}K_0(N r'_w)}\cos(n\pi z/b)K_0(N r')\end{aligned}. \quad \text{(A14)}$$

Recalling that $r' = rK_D^{1/2}$ and rewriting (A14) in dimensionless form yields (6).

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REFERENCES

Churchil, R.V. (1958), *Operational Mathematics*, McGraw Hill, New York.

Crump, K. S. (1976), Numerical inversion of Laplace transforms using a Fourier series approximation, *J. Assoc. Comput. Mach.*, *23*, 89–96.

de Hoog, F. R., J. H. Knight, and A. N. Stokes (1982), An improved method for numerical inversion of Laplace transforms, *SIAM J. Sci. Comput.*, *3*, 357–366, doi:10.1137/0903022.

Hantush, M. S. (1964), Hydraulics of wells, *Adv. Hydrosci.*, *1*, 281–442.

Mathias, S. A., and A. P. Butler (2006), Flow to a finite diameter well in a horizontally anisotropic aquifer with wellbore storage, *Water Resour. Res.*, *43*, W07501, doi: 10.1029/2006WR005839.

Theis, C. V. (1935), The relationship between the lowering of the piezometric surface and rate and duration of discharge of a well using groundwater storage, *Eos Trans. AGU*, *16*, 519–524.

Yang, S.-Y., H.-D. Yeh, and P.-Y. Chiu (2006), A closed form solution for constant flux pumping in a well under partial penetration condition, Water Resour. Res., 42, W05502, doi:10.1029/2004WR003889.

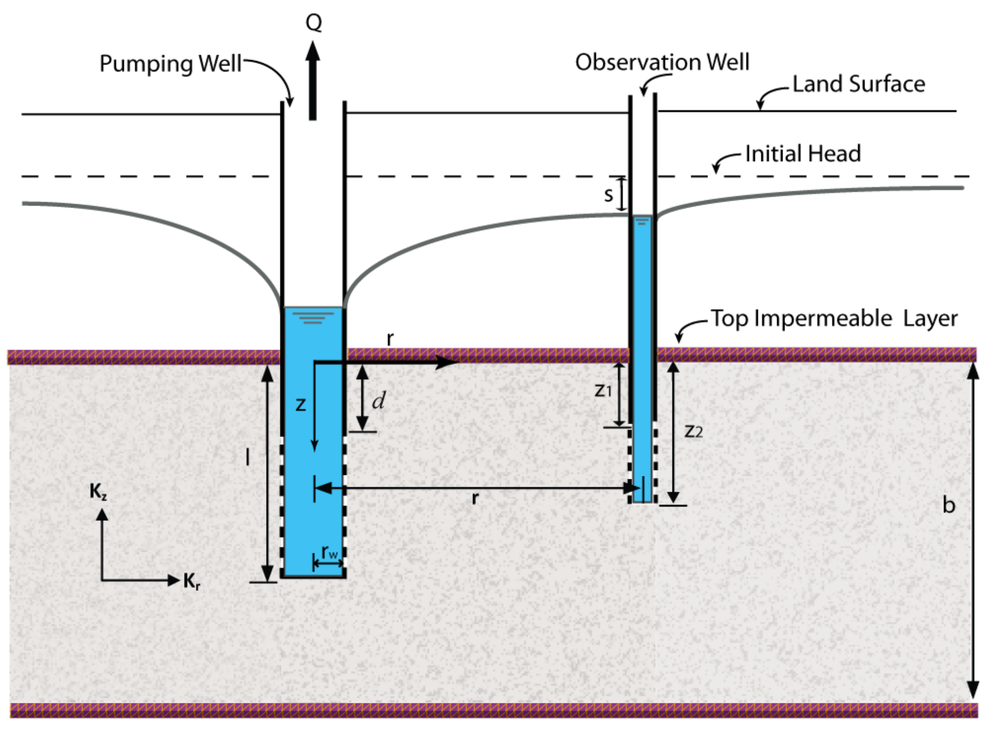

Figure 1: Schematic representation of system geometry

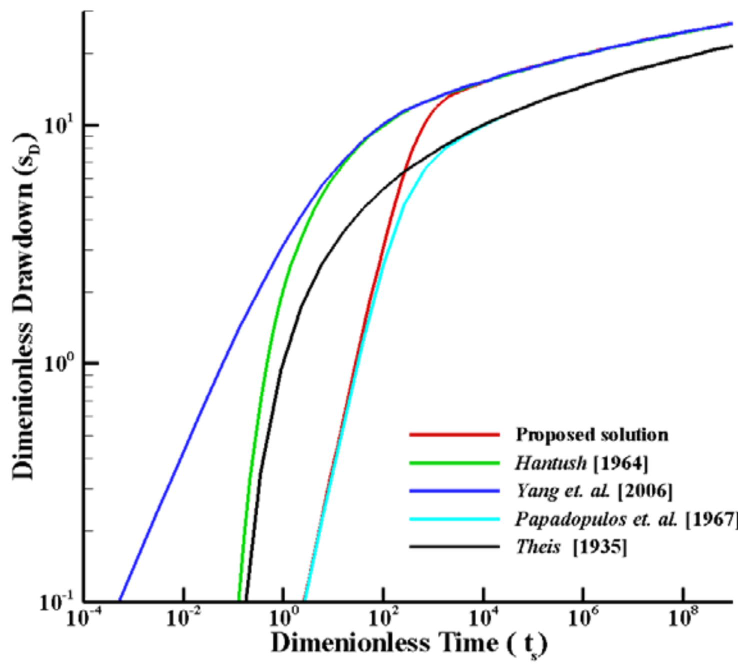

Figure 2: Dimensionless drawdown in pumping well versus dimensionless time, computed by various analytical solutions when $C_{wD} = 1.0 \times 10^2$, $d_D = 0.0$, $l_D = 0.5$ and $K_D = 1.0$.

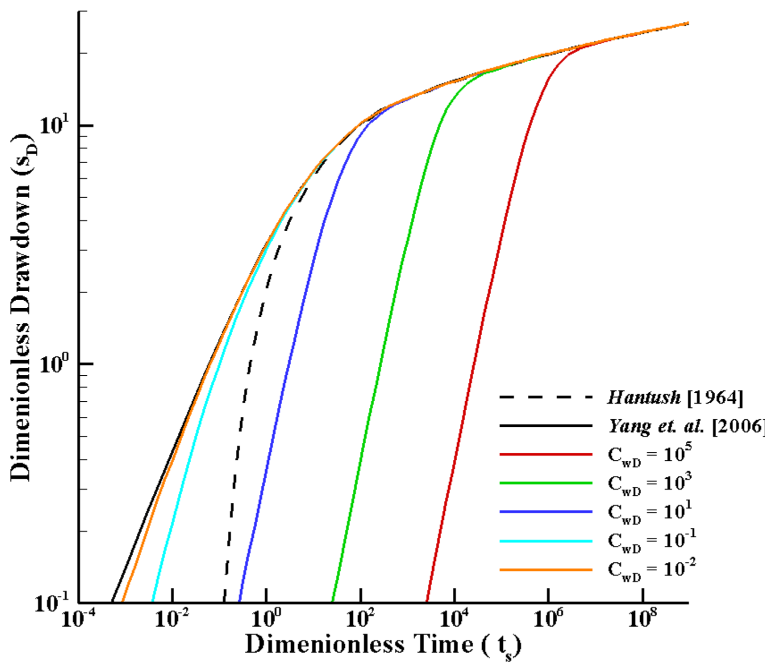

Figure 3: Dimensionless drawdown in pumping well versus dimensionless time for various values of dimensionless wellbore storage C_{wD}, when $d_D = 0.0$, $l_D = 0.5$ and $K_D = 1.0$.

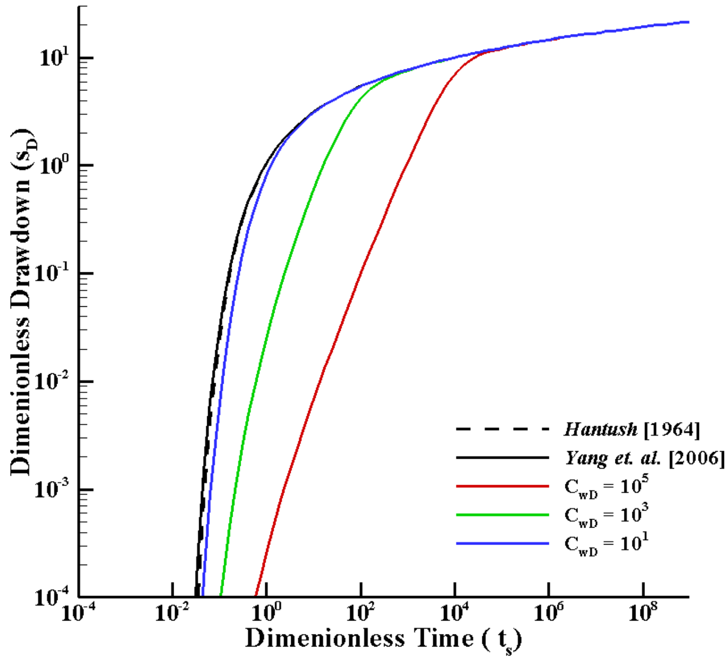

Figure 4: Dimensionless drawdown at $z/b = 0.5$, $r/b = 0.2$ versus dimensionless time for various values of dimensionless wellbore storage C_{wD} , when $l_D = 0.5$ and $K_D = 1.0$.

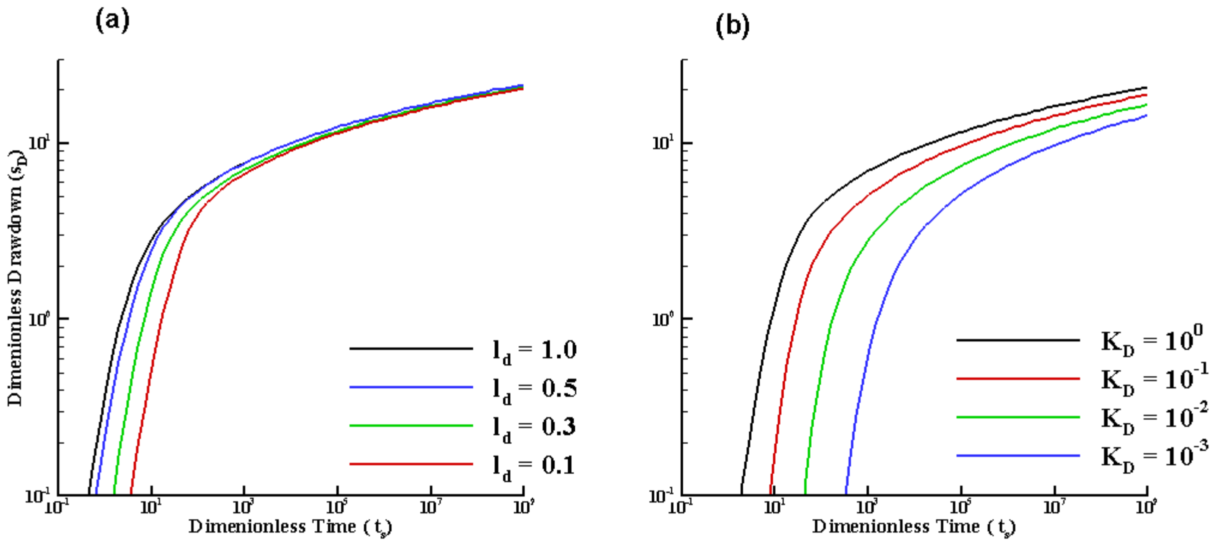

Figure 5: Dimensionless drawdown in pumping well versus dimensionless time for various (a) screen lengths l_D when $d_D = 0$ and $C_{wD} = 1.0 \times 10^2$ (b) anisotropy ratio $K_D = K_z / K_r$ when $C_{wD} = 1.0 \times 10^2$, $d_D = 0.0$ and $l_D = 0.25$.

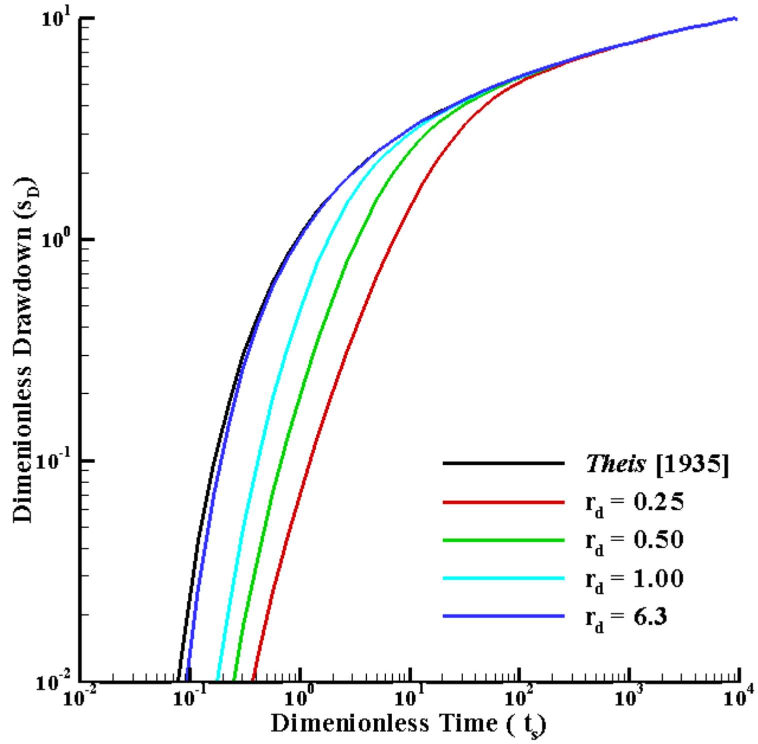

Figure 6: Dimensionless drawdown at observation piezometer versus dimensionless time for various values of dimensionless radial distance $r_D = r/b$ when $C_{wD} = 1.0 \times 10^3$, $K_D = 1.0$, $d_D = 0.0$, $l_D = 0.25$ and $z/b = 0.5$