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Technical Note

Radial flow to a partially penetrating well with storage in an anisotropic confined aquifer

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SUMMARY

Drawdowns generated by extracting water from large diameter (e.g. water supply) well are affected by wellbore storage. We present an analytical solution in Laplace transformed space for drawdown in a uniform anisotropic aquifer caused by withdrawing water at a constant rate from partially penetrating well with storage. The solution is back transformed into the time domain numerically. When the pumping well is fully penetrating our solution reduces to that of Papadopulos and Cooper (1967); Hantush (1964) when the pumping well has no wellbore storage; Theis (1935) when both conditions are fulfilled and Yang (2006) when the pumping well is partially penetrating, has finite radius but lacks storage. Newly developed solution is then used to explore graphically the effects of partial penetration, wellbore storage and anisotropy on time evolutions of drawdown in the pumping well and in observation wells. We concluded after validating the developed analytical solution using synthetic pumping test.

1. Introduction

When water is pumped from a large diameter (e.g. water supply) well drawdown in the surrounding aquifer is affected by temporal decline in wellbore storage. An analytical solution accounting for this effect under radial flow toward a fully penetrating well of finite diameter with storage was developed by Papadopulos and Cooper (1967). A corresponding solution without wellbore storage was presented earlier by van Everdingen and Hurst (1949) and later, in elliptical coordinates, by Kucuk and Brigham (1979). Mathias and Butler (2007) extended the solution of Kucuk and Brigham (1979) by adding wellbore storage and horizontal anisotropy. Their solution utilized Mathieu functions in Laplace transformed space and numerical inversion of the result into the time domain. Yang (2006) extended the solution of van Everdingen and Hurst (1949) by allowing the pumping well to be partially penetrating. Dougherty and Babu (1984) developed an analytical solution for a pumping well with storage in a confined double porosity reservoir. Their solution can be reduced to that for a single porosity confined aquifer but ignores anisotropy. None of the available analytical solutions account simultaneously for aquifer anisotropy, partial penetration and storage capacity of the pumping well under confined aquifer conditions.

Moench (1997, 1998) developed an analytical solution for flow to a pumping well with storage in a uniform anisotropic unconfined (water table) aquifer. Following similar approach, we present a new solution for radial flow to a partially penetrating well of finite diameter with storage in an anisotropic confined aquifer. Whereas Moench (1997, 1998) used Fourier cosine series in Laplace transformed space we employ Laplace transformation with respect to time followed by finite cosine transformation with respect to vertical coordinates. The presented solution reduces to that of Papadopulos and Cooper (1967) when the pumping well is fully penetrating, Hantush (1964) in the absence of wellbore storage, Theis (1935) when both conditions are fulfilled, and Yang (2006) when the pumping well is partially penetrating, has finite radius but lacks storage. We use our solution to explore graphically the effects of partial penetration, wellbore storage and anisotropy on time evolutions of drawdown in the pumping well and in observation wells.

2. Theory

2.1. Problem definition

Consider a well of finite radius r_w that is in hydraulic contact with a surrounding confined aquifer at depths d through l below

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the top (Fig. 1). The aquifer is horizontal and of infinite lateral extent with uniform thickness b, uniform hydraulic properties and anisotropy ratio $K_D = K_z/K_r$ between vertical and horizontal hydraulic conductivities, K_z and K_r, respectively.

Initially, drawdown $s(r,z,t)$ throughout the aquifer is zero where r is radial distance from the axis of the well, z is depth below the top of the aquifer and t is time. Starting at time $t=0$ water is withdrawn from the pumping well at a constant volumetric rate Q. Consider the bottom of the well to be impermeable and ignore flow beneath it. Then drawdown distribution in space–time is controlled by

$$K_r\left(\frac{\partial^2 s}{\partial r^2}+\frac{1}{r}\frac{\partial s}{\partial r}\right)+K_s\frac{\partial^2 s}{\partial z^2}=S_s\frac{\partial s}{\partial t}\quad 0\leqslant z\leqslant b \tag{1}$$

along with initial and far-field boundary condition

$$s(r,z,0)=s(\infty,z,0)=0\quad r\geqslant r_w, \tag{2}$$

no-flow condition at top and bottom impermeable boundaries

$$\frac{\partial s}{\partial z}=0\ z=0 \text{ and } z=b,\quad r>r_w, \tag{3}$$

the wellbore storage mass-balance expression assuming constant flux across screen (see Zlotnik and Zhan, 2005 for discussion of this assumption's validity)

$$2\pi(l-d)K_r r_w\left(\frac{\partial s}{\partial r}\right)_{r=r_w}-C_w\left(\frac{\partial s}{\partial t}\right)_{r=r_w}=-Q\quad d<z<l, \tag{4}$$

and the no-flow condition at the portion of the well casing that is not open to the aquifer

$$r\left(\frac{\partial s}{\partial r}\right)=0\ 0<z<d\ \&\ l<z<b \tag{5}$$

where C_w is wellbore storage coefficient (volume of water released from well storage per unit drawdown in it).

2.2. Solution in Laplace space

We show in Appendix A that the Laplace transform of the solution, indicated by an overbar, is given by

$$\begin{aligned}\bar{s}(r_D,z_D,p_D)=\frac{Qt}{4\pi K_r b}\Bigg\{&\frac{2}{p_D}\frac{K_0(\phi_0)}{r_{wD}\phi_0 K_1(r_{wD}\phi_n)+\frac{C_{wD}}{2(l_D-d_D)}r_{wD}^2\phi_0^2K_0(r_{wD}\phi_0)}\\&+\frac{4}{p_D\pi(l_D-d_D)}\sum_{n=1}^{\infty}\\&\times\frac{K_0(\phi_n)[sin(n\pi l_D)-sin(n\pi d_D)]cos(n\pi z_D)/n}{r_{wD}\phi_n K_1(r_{wD}\phi_n)+\frac{C_{wD}}{2(l_D-d_D)}r_{wD}^2\phi_0^2K_0(r_{wD}\phi_n)}\Bigg\}\end{aligned} \tag{6}$$

where
$r_D=r/b, z_D=z/b, r_{wD}=r_w/r, C_{wD}=C_w/(\pi S_s r_w^2), d_D=d/b, l_D=l/b$, $\phi_n=\sqrt{p_D/t_D+\beta^2n^2\pi^2}, t_D=\alpha_s t/r^2, \alpha_s=K_r/S_s, S_s$ is specific storage, $\beta=r_D K_D^{1/2}, p_D=pt, p$ is Laplace parameter, K_0 and K_1 are second-kind modified Bessesl functions of order zero and one, respectively. A corresponding solution in time domain $s(r_D,z_D,t_D)$, is obtained through numerical inversion of the Laplace transform by means of an algorithm due to Crump, 1976 as modified by de Hoog et al., 1982. Whereas standard inversion with respect to p is done over a time interval $[0,t]$, here the inversion is performed with respect to p_D over a unit dimensionless time (corresponding to p_D^{-1}) interval $[0,1]$, regardless of what t_D is.

2.3. Vertically averaged drawdwon

Drawdown in a piezometer or observation well that penetrates the aquifer between dimensionless depths $z_{D1}=z_1/b$ and $z_{D2}=z_2/b$ at a dimensionless radial distance r_D from the pumping well (Fig. 1) is obtained by averaging the point drawdown over this interval according to

$$\bar{s}_{z_{D2}-z_{D1}}(r_D,p_D)=\frac{1}{z_{D2}-z_{D1}}\int_{z_{D1}}^{z_{D2}}s(r_D,z_D,p_D)\mathrm{d}z_D. \tag{7}$$

Substituting (6) into (7) and evaluating the integral gives

$$\begin{aligned}\bar{s}_{z_{D2}-z_{D1}}(r_D,p_D)=\frac{Qt}{4\pi K_r b}\Bigg\{&\frac{2}{p_D}\frac{K_0(\phi_0)}{r_{wD}\phi_0K_1(r_{wD}\phi_n)+\frac{C_{wD}}{2(l_D-d_D)}r_{wD}^2\phi_0^2K_0(r_{wD}\phi_0)}\\&+\frac{4}{p_D\pi^2(l_D-d_D)(z_{D2}-z_{D1})}\sum_{n=1}^{\infty}\frac{K_0(\phi_n)}{n^2}\\&\times\frac{[sin(n\pi l_D)-sin(n\pi d_D)][sin(n\pi z_{D2})-sin(n\pi z_{D1})]}{r_{wD}\phi_nK_1(r_{wD}\phi_n)+\frac{C_{wD}}{2(l_D-d_D)}r_{wD}^2\phi_0^2K_0(r_{wD}\phi_n)}\Bigg\}\end{aligned} \tag{8}$$

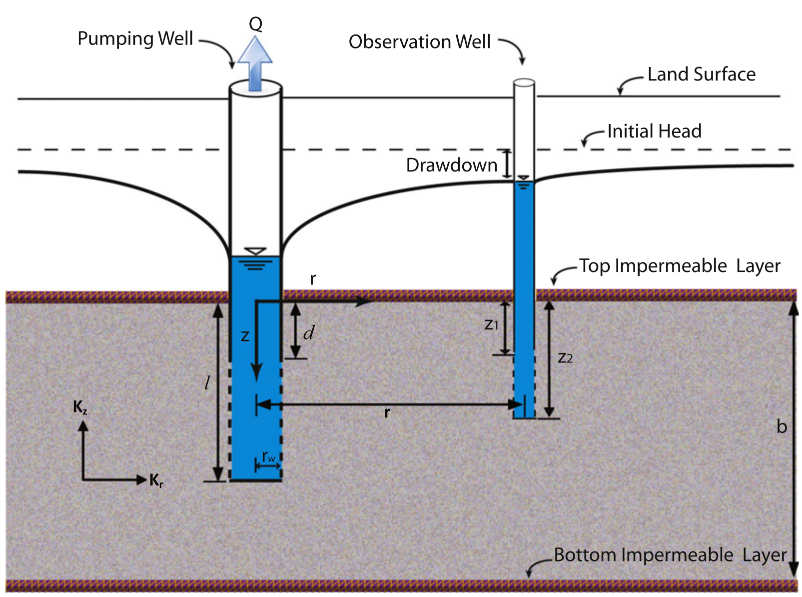

Fig. 1. Schematic representation of confined aquifer geometry.

3. Reduction to other solutions

3.1. Reduction to solution of Papadopulos and Cooper (1967)

When the pumping well is fully penetrating $l_D = 1, d_D = 0$ and (6) reduces to the corresponding Laplace domain solution of Papadopulos and Cooper (1967),

$$\bar{s}(r_D, p_D) = \frac{Qt}{4\pi K_r b}\left\{\frac{2}{p_D} \times \frac{K_0(\phi_0)}{r_{wD}\phi_0 K_1(r_{wD}\phi_0) + \frac{C_{wD}}{2} r_{wD}^2 \phi_0^2 K_0(r_{wD}\phi_0)}\right\} \quad (9)$$

3.2. Reduction to solution's of Yang (2006); Hantush (1964) and Theis (1935)

When the pumping well has finite diameter ($r_w \neq 0$) but negligible or no wellbore storage ($C_{wD} \to 0$), (6) reduces to the solution of Yang (2006) in Laplace space,

$$\bar{s}(r_D, z_D, p_D) = \frac{Qt}{4\pi K_r b}\left\{\frac{2}{p_D}\frac{K_0(\phi_0)}{r_{wD}\phi_0 K_1(r_{wD}\phi_0)} + \frac{4}{p_D \pi (l_D - d_D)}\sum_{n=1}^{\infty} \times \frac{K_0(\phi_0)[\sin(n\pi l_D) - \sin(n\pi d_D)]\cos(n\pi z_D)}{n r_{wD}\phi_n K_1(r_{wD}\phi_n)}\right\} \quad (10)$$

When the pumping well has small diameter ($r_w \to 0$), (6) reduces to Hantush (1964) solution in Laplace space due to the fact that $xK_1(x) \to 1$ and $x^2 K_0(x) \to 0$ as $\to 0$,

$$\bar{s}(r_D, z_D, p_D) = \frac{Qt}{4\pi K_r b}\left\{\frac{2}{p_D}K_0(\phi_0) + \frac{4}{p_D \pi}\sum_{n=1}^{\infty} \times \frac{K_0(\phi_0)[\sin(n\pi l_D) - \sin(n\pi d_D)]\cos(n\pi z_D)}{n(l_D - d_D)}\right\} \quad (11)$$

It is well established and easily verified that the latter in turn reduces to the Theis (1935) solution in Laplace space when the pumping well becomes fully penetrating ($d_D = 0, l_D = 1$),

$$\bar{s}(r_D, z_D, p_D) = \frac{Qt}{4\pi K_r b}\left\{\frac{2}{p_D}K_0(\phi_0)\right\} \quad (12)$$

4. Results and discussion

To investigate the effect of partial penetration, wellbore storage and anisotropy on drawdown we consider a pumping well of dimensionless radius $r_w/b = 0.02$.

4.1. Drawdown in pumping well

We start by considering drawdown in a pumping well ($r_D = r_w/b$) penetrating the upper half ($d_D = 0, l_D = 1$) of an isotropic aquifer with $K_D = 1$. Fig. 2 compares the variation of dimensionless drawdown $s_D(r_D, z_D, t_D) = (4\pi K_r b/Q)s(r_D, z_d, t_D)$ in the pumping well with dimensionless time t_D using different analytical solutions when $C_{wD} = 1.0 \times 10^2$. At early time water is derived entirely from wellbore storage, rendering dimensionless drawdown linearly proportional to dimensionless time (forming a line with unit slope on log–log scale); solution presented here and that of Papadopulos and Cooper (1967) reflect this clearly. Solutions that do not account for wellbore storage predict a much earlier rise in drawdown. Whereas the Papadopulos and Cooper (1967) solution approaches that of Theis (1935) at later dimensionless time, presented solution approaches that of Hantush (1964) as the effects of finite radius and wellbore storage dissipate. The solution of Yang (2006), which considers only the first effect, exhibits an earlier rise in dimensionless drawdown than do any of the other solutions, eventually coinciding with that of Hantush (1964). Dimensionless drawdown in the pumping well at late dimensionless time exceeds that predicted by solutions which ignore partial penetration.

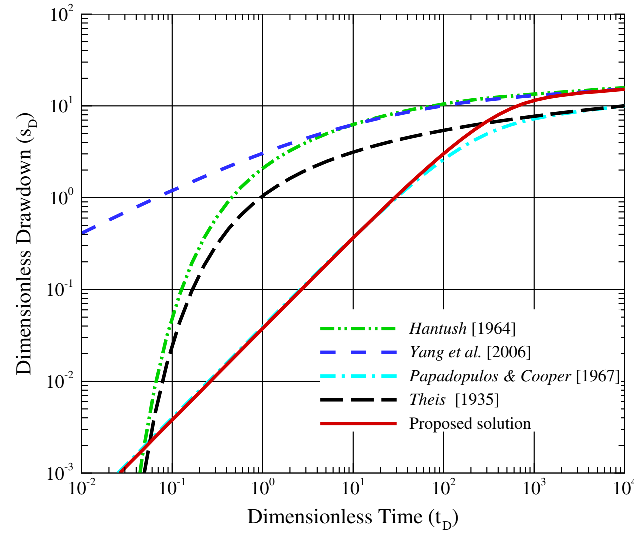

Fig. 2. Dimensionless drawdown in pumping well versus dimensionless time, computed by various analytical solutions when $C_{wD} = 1.0 \times 10^2, d_D = 0.0, l_D = 0.5$ and $K_D = 1.0$.

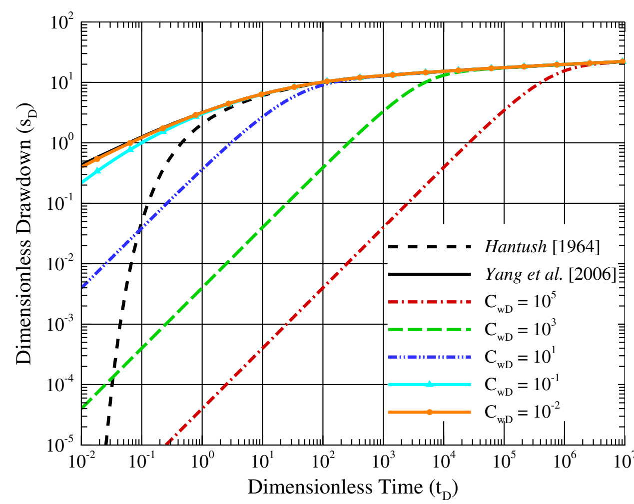

Fig. 3. Dimensionless drawdown in pumping well versus dimensionless time for various values of dimensionless wellbore storage C_{wD} when $d_D = 0.0, l_D = 0.5$ and $K_D = 1.0$.

Fig. 3 shows how dimensionless drawdown in the pumping well varies with dimensionless time t_D for different values of the dimensionless wellbore storage coefficient, C_{wD}. As with the solution of Papadopulos and Cooper (1967), the larger is C_{wD} the longer does wellbore storage impact drawdown in the pumping well. As C_{wD} diminishes presented solution approaches that of Yang (2006).

4.2. Drawdown in piezometer

Fig. 4 shows dimensionless time-drawdown variations at dimensionless radial distance $r_D = 0.2$ from the axis of the pumping well and dimensionless elevation $z_D = 0.5$ (midway between the horizontal no-flow boundaries) for different values of C_{wD} under the above conditions. When C_{wD} is large, the early dimensionless time-drawdown curve on log–log scale is nearly linear

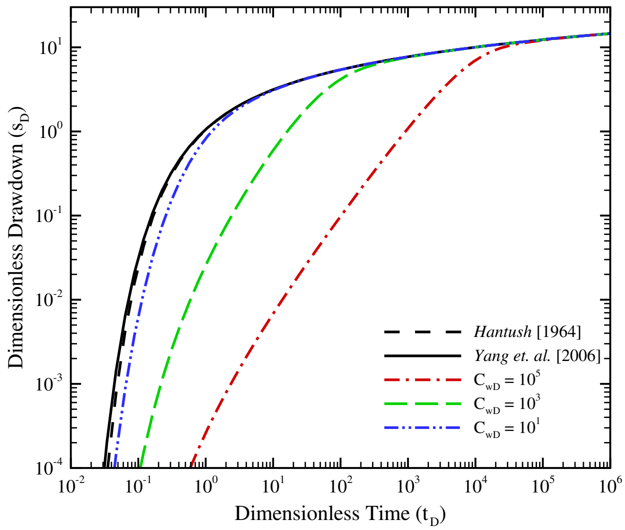

Fig. 4. Dimensionless drawdown at $z_D = 0.5$ and $r_D = 0.2$ versus dimensionless time for various values of dimensionless wellbore storage C_{wD} when $l_D = 0.5$ and $K_D = 1.0$.

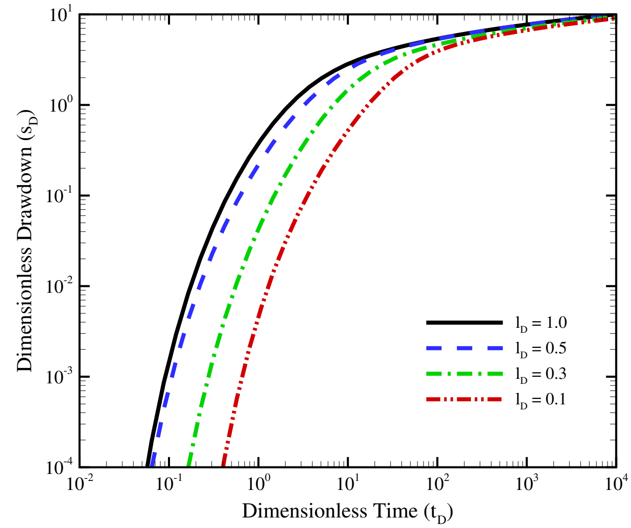

Fig. 6. Dimensionless drawdown at $z_D = 0.5$ and $r_D = 0.2$ versus dimensionless time for various anisotropy ratios $K_D = K_z/K_r$ when $C_{wD} = 1.0 \times 10^2, d_D = 0.0$ and $l_D = 0.25$.

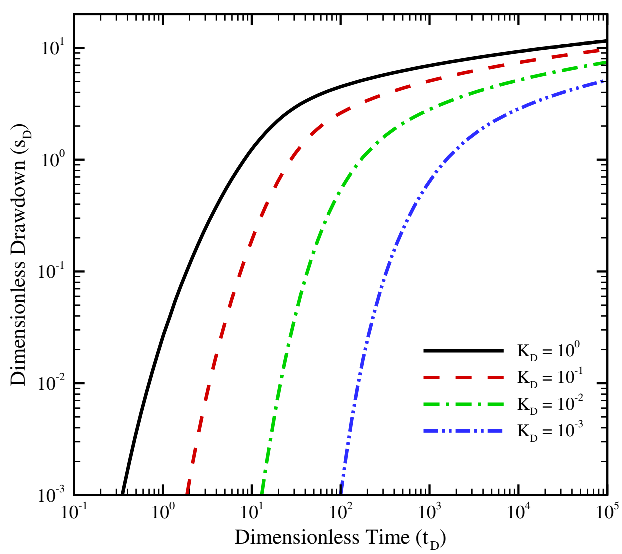

Fig. 5. Dimensionless drawdown at $z_D = 0.5$ and $r_D = 0.2$ versus dimensionless time for various screen lengths l_D when $C_{wD} = 1.0 \times 10^2, d_D = 0$ and $K_D = 1.0$.

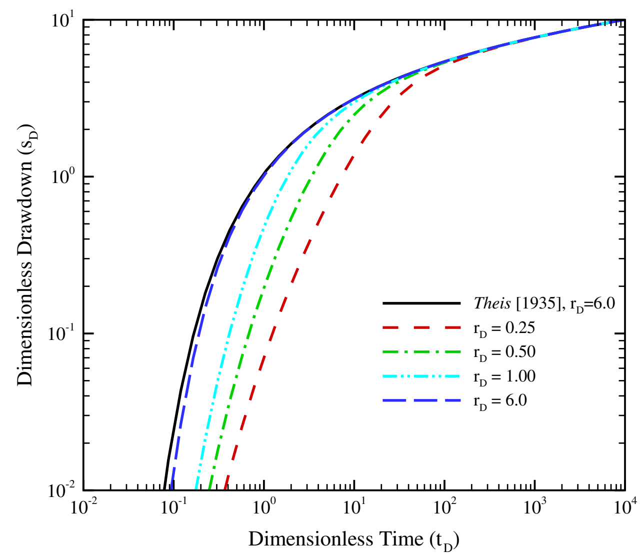

Fig. 7. Dimensionless drawdown versus dimensionless time at $z_D = 0.5$ and various values of $r_D = r/b$ when $C_{wD} = 1.0 \times 10^3, d_D = 0.0, l_D = 0.25$ and $K_D = 1.0$.

with a unit slope, reflecting a strong effect of storage in the pumping well on early drawdown in a nearby piezometer. As C_{wD} diminishes this effect becomes less discernible, the curve becoming nonlinear and steeper. The curve tends asymptotically toward the solution of Yang (2006), which in turn is very close to that of Hantush (1964) due to the small dimensionless radius we have assigned to the pumping well in our example.

Fig. 5, corresponding to the case where $C_{wD} = 1.0 \times 10^2$, shows that dimensionless drawdown at $r_D = 0.2$ and $z_D = 0.5$ increases when the pumping well is extended to the aquifer bottom ($d_D = 0.0,\ l_D = 1.0$ below the observation point) but decreases when this well becomes shallower; a similar trend is reflected in the solution of Hantush (1964). Reducing the ratio K_D between vertical and horizontal hydraulic conductivity in the case of a well that is shallower than the observation point ($d_D = 0.0,\ l_D = 0.25$) likewise causes dimensionless drawdown at this point to diminish (Fig. 6).

Fig. 7 illustrates the impact of dimensionless radial distance from the pumping well on dimensionless time-drawdown at $z_D = 0.5$ when $d_D = 0.0,\ l_D = 0.25, K_D = 1$ and $C_{wD} = 1.0 \times 10^2$. As this distance increases the effects of both wellbore storage and partial penetration diminish, the dimensionless time-drawdown response in the aquifer approaching that predicted by Theis (1935).

5. Summary and conclusion

A new analytical solution has been developed for a partially penetrating well of finite diameter with storage pumping at a constant rate from an anisotropic confined aquifer. Our solution unifies the solutions of Papadopulos and Cooper (1967); Hantush (1964); Theis (1935) and Yang (2006) by accounting simultaneously for aquifer anisotropy, partial penetration and wellbore storage capacity of the pumping well under confined conditions.

We used our solution to explore all three effects. Reducing the anisotropy ratio causes drawdown in the aquifer to decrease. Whereas the effect of partial penetration decreases with increasing distance from the pumping well, that of wellbore storage diminishes with distance and time.

Appendix A. Laplace transformed drawdown for confined aquifer

Introducing a new variable $r' = r(K_z/K_r)^{1/2} = rK_D^{1/2}$ and taking Laplace transform (1)–(5) gives

$$\frac{\partial^2 \bar{s}}{\partial r'^2} + \frac{1}{r'}\frac{\partial \bar{s}}{\partial r'} + \frac{\partial^2 \bar{s}}{\partial z^2} = \frac{S_s}{K_z} p\bar{s} \quad 0 \leqslant z < b \tag{A.1}$$

subject to

$$\bar{s}(\infty, z, p) = 0 \tag{A.2}$$

$$\frac{\partial \bar{s}}{\partial z} = 0 \quad \text{at } z = 0 \text{ and } z = b \quad r > r_w \tag{A.3}$$

$$2\pi(l-d)K_r r'_w \left(\frac{\partial \bar{s}}{\partial r'}\right)_{r\prime = r'_w} - C_w p\bar{s}_{r'=r'_w} = -\frac{Q}{p} \quad d < z < l \tag{A.4}$$

$$r'_w \left(\frac{\partial \bar{s}}{\partial r'}\right)_{r'=r'_w} = 0 // \text{for} \quad 0 \leqslant z < d \text{ and } l < z \leqslant b \tag{A.5}$$

Defining the finite cosine transform of $\bar{s}(r', z, p)$ as (Churchill, 1958, p. 354–355)

$$f_c\{(\bar{s}(r', z, p)\} = \bar{s}_c(r', n, p) = \int_0^b \bar{s}(r', z, p) cos(n\pi z/b) dz \quad n = 0, 1, 2 \ldots \tag{A.6}$$

with inverse

$$\bar{s}(r', z, p) = \frac{1}{b}\bar{s}_c(r', 0, p) + \frac{2}{b}\sum_{n=1}^{\infty} \bar{s}_c(r', n, p) cos(n\pi z/b) \tag{A.7}$$

implies that, by virtue of (A.3),

$$f_c\left\{\frac{\partial^2 \bar{s}}{\partial z^2}\right\} = -\frac{n^2\pi^2}{b^2}\bar{s}_c(r', n, p) + (-1)^n \frac{\partial \bar{s}(r', z, p)}{\partial z}|_{z=b} - \frac{\partial \bar{s}(r', z, p)}{\partial z}|_{z=0} = -\left(\frac{n\pi}{b}\right)^2 \bar{s}_c(r', n, p) \tag{A.8}$$

Hence finite cosine transformation of (A.1)–(A.5) leads to

$$\frac{\partial^2 \bar{s}_c}{\partial r'^2} + \frac{1}{r}\frac{\partial \bar{s}_c}{\partial r'} - \left[\frac{p}{K_z/S_s} + \left(\frac{n\pi}{b}\right)^2\right]\bar{s}_c = 0 \tag{A.9}$$

$$\bar{s}_c(\infty, n, p) = 0 \tag{A.10}$$

$$2\pi(l-d)K_r r'_w \left(\frac{\partial \bar{s}}{\partial r'}\right)_{r\prime = r'_w} - C_w p\bar{s}_{r'=r'_w} = -\frac{Q}{p}\int_l^d cos(n\pi z/b) dz = -\frac{Q}{p}(b/n\pi)[sin(n\pi l/b) - sin(n\pi d/b)] \tag{A.11}$$

The general solution of (A.9) is

$$\bar{s}_c(r', n, p) = AK_0(Nr') + BI_0(Nr') \tag{A.12}$$

where $N^2 = \frac{p}{K_z/S_s} + (n\pi/b)^2$, I_0 and K_0 being modified Bessel functions of first and second kind, respectively, and of zero order. By virtue of (A.10) $B = 0$. Substituting this and (A.12) into (A.11), noting that $\partial K_0(Nr')/\partial r' = -NK_1(Nr')$, solving for A and substituting back into (A.11) yields

$$\bar{s}_c(r', n, p) = \frac{Q}{p}\frac{(b/n\pi)[sin(n\pi l/b) - sin(n\pi d/b)]}{2\pi(l-d)K_r N r'_w K_1(Nr'_w) + pC_w K_0(Nr'_w)} K_0(Nr') \tag{A.13}$$

Noting that

$$\lim_{n\to 0}\left[l\frac{sin(n\pi l/b)}{n\pi l/b} - d\frac{sin(n\pi d/b)}{n\pi d/b}\right] = l - d \tag{A.14}$$

one gets

$$\bar{s}_c(r', 0, p) = \frac{Q}{p}\frac{K_0\left(r'\sqrt{\frac{p}{K_z/S_s}}\right)}{2\pi K_r K_1\left(r'_w\sqrt{\frac{p}{K_z/S_s}}\right) + \frac{pC_w}{l-d}K_0\left(r'_w\sqrt{\frac{p}{K_z/S_s}}\right)} \tag{A.15}$$

This allows obtaining the inverse Fourier cosine transform of (A.13),

$$\bar{s}(r', z, p) = \frac{1}{b}\frac{Q}{K_r p}\frac{K_0\left(r'\sqrt{\frac{p}{K_z/S_s}}\right)}{2\pi r'_w\sqrt{\frac{p}{K_z/S_s}}K_1\left(r'_w\sqrt{\frac{p}{K_z/S_s}}\right) + \frac{pC_w}{K_r(l-d)}K_0\left(r'_w\sqrt{\frac{p}{K_z/S_s}}\right)} + \frac{2}{b}\frac{Q}{K_r p}\sum_{n=0}^{\infty}\frac{(b/n\pi)[sin(n\pi l/b) - sin(n\pi d/b)]}{2\pi(l-d)K_r N r'_w K_1(Nr'_w) + pC_w K_0(Nr'_w)} cos(n\pi z/b) K_0(Nr') \tag{A.16}$$

Recalling that $r' = rK_D^{1/2}$ and rewriting (A.16) in dimensionless form yields (6).

References

Churchill, R., 1958. Operational Mathematics. McGraw-Hill, New York.

Crump, K., 1976. Numerical inversion of Laplace transforms using a Fourier series approximation. Journal of the ACM (JACM) 23 (1), 89–96.

de Hoog, F., Knight, J., Stokes, A., 1982. An improved method for numerical inversion of laplace transforms. SIAM Journal on Scientific and Statistical Computing 3, 357.

Dougherty, D., Babu, D., 1984. Flow to a partially penetrating well in a double-porosity reservoir. Water Resources Research 20 (8), 1116–1122.

Hantush, M., 1964. Hydraulics of wells. Advances in hydroscience 1, 281–432.

Kucuk, F., Brigham, W., 1979. Transient flow in elliptical systems. SPE Journal 19 (6), 401–410.

Mathias, S., Butler, A., 2007. Flow to a finite diameter well in a horizontally anisotropic aquifer with wellbore storage. Water Resources Research 43 (7), W07501.

Moench, A., 1997. Flow to a well of finite diameter in a homogeneous, anisotropic water table aquifer. Water Resources Research 33 (6), 1397–1407.

Moench, A., 1998. Correction to flow to a well of finite diameter in a homogeneous, anisotropic water table aquifer by Allen F. Moench. Water Resources Research 34 (9), 2431–2432.

Papadopulos, I., Cooper, H., 1967. Drawdown in a well of large diameter. Water Resources Research 3 (1), 241–244.

Theis, C., 1935. The relation between the lowering of the piezometric surface and the rate and duration of discharge of a well using groundwater storage. Transactions of American Geo- physical Union 16 (1), 519–524.

van Everdingen, A., Hurst, W., 1949. The application of the Laplace transformation to flow problems in reservoirs. Transformation, 305–324.

Yang, S., Yeh, H.-D., Chiu, P., 2006. A closed form solution for constant flux pumping in a well under partial penetration condition. Water Resources Research 42 (5), W05502.

Zlotnik, V., Zhan, H., 2005. Aquitard effect on drawdown in water table aquifers. Water Resources Research 41 (6), W06022.